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MODULE 1
PROBLEM SOLVING

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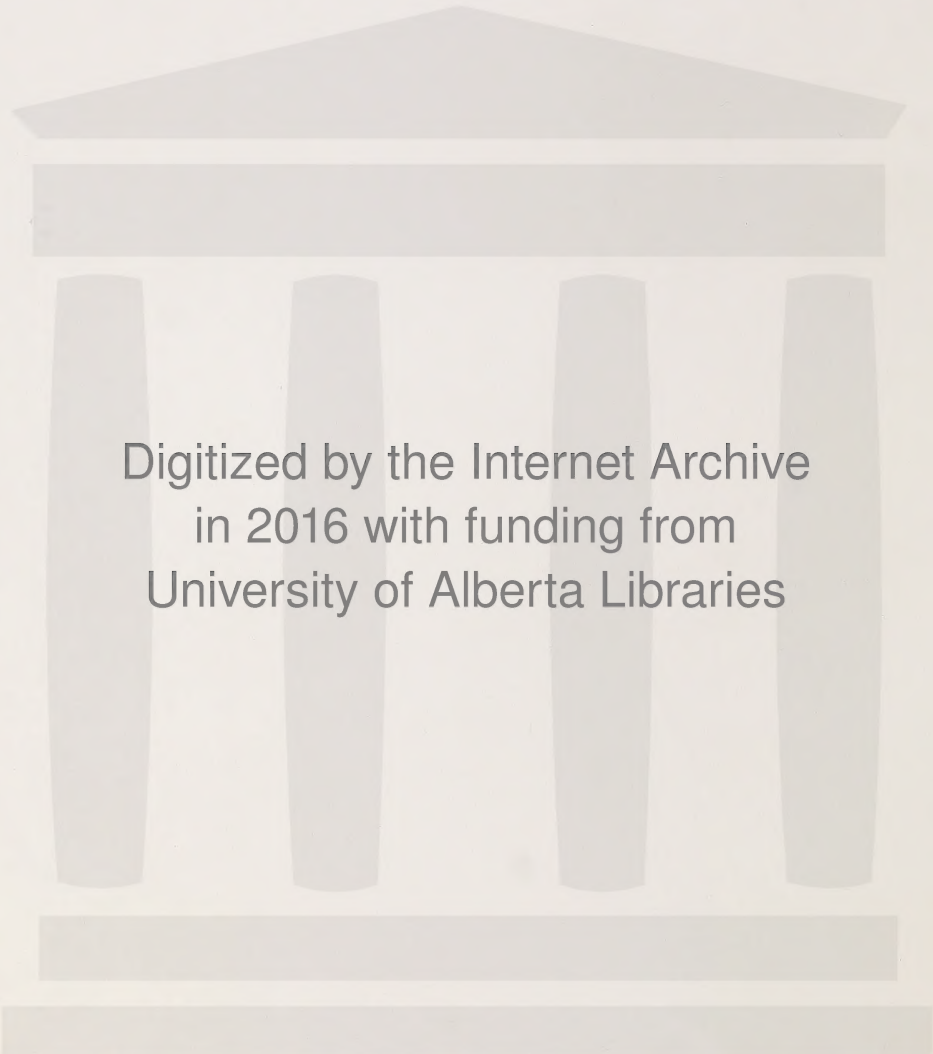
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Mathematics 8

Module 1: Problem Solving

MODULE BOOKLET

Mathematics 8
Student Module
Module 1
Problem Solving

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Welcome to Mathematics 8!

We hope you'll find this course interesting and fun.

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,

turn on your audiocassette and listen.

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MODULE INTRODUCTION

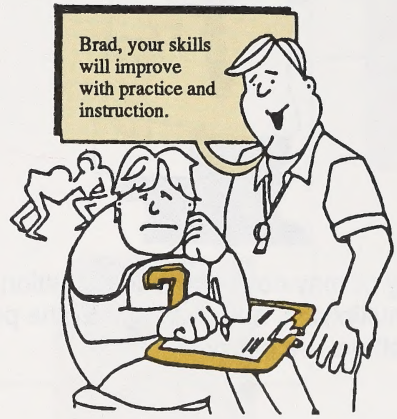
What Lies Ahead

In this Module Introduction you will preview Module 1 and learn how it will be evaluated.

Working Together

One of the main goals of this mathematics course is to improve your problem-solving skills. You will need these problem-solving skills many times in your lifetime.

Like any skill, the skill of problem solving must be developed.



This module will give you hints about problem solving. You will have many opportunities to practise solving problems in this module and in the other modules of the course.

Problem Solving

Since this module focuses on problem solving, it is important that you understand what a problem is. A **problem** is a task for which the method for finding the answer (as well as the answer) is not immediately known.

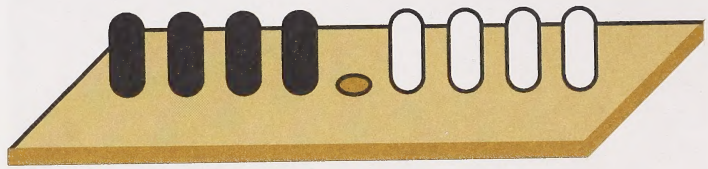
Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

Example 1: Realistic Problem

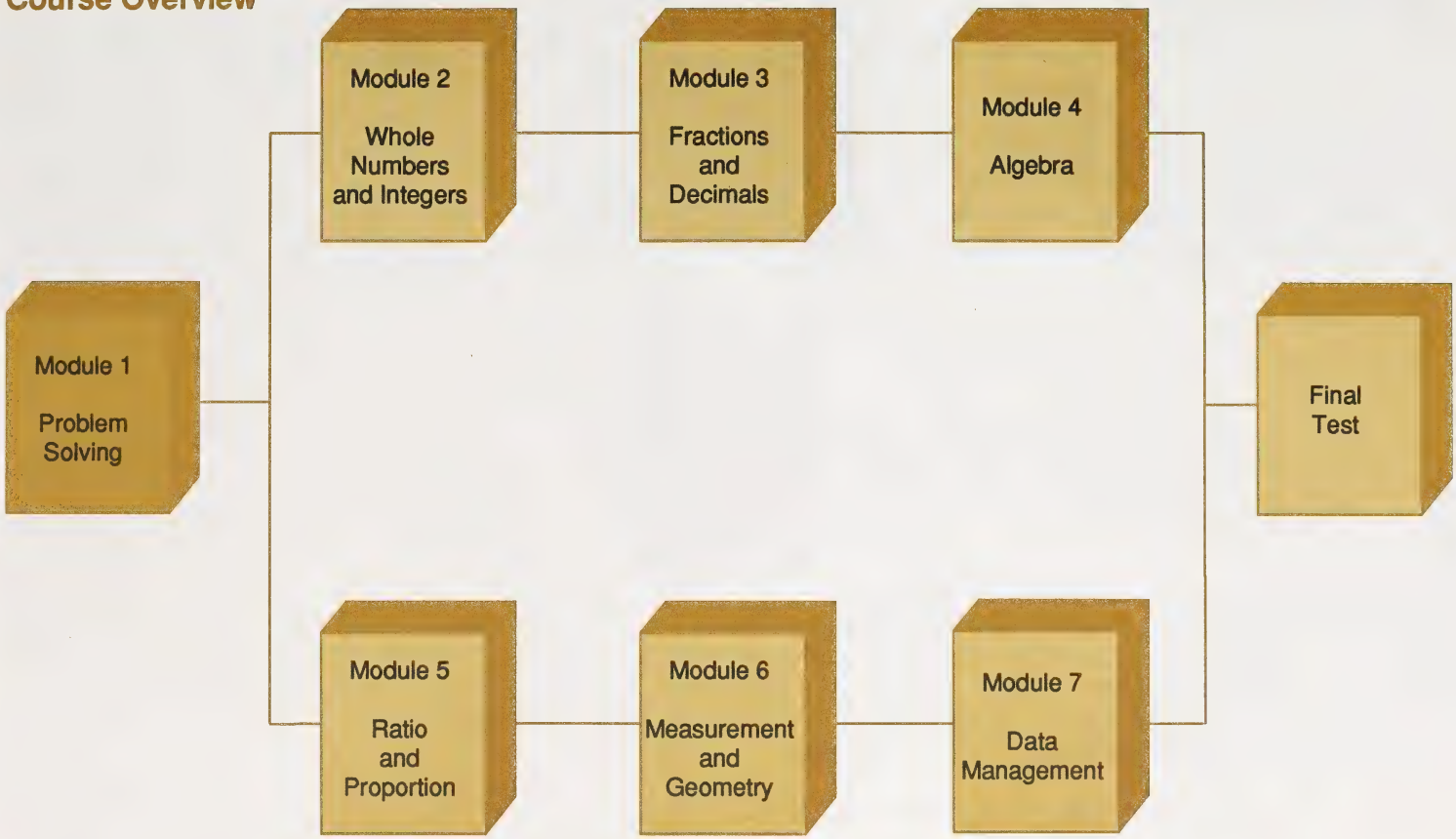
Thirty-two students are competing in a singles badminton tournament. When a student loses, that student is out of the tournament. How many games must be played before there is a winner?

Example 2: Puzzle Problem

The object of a game is to move all the pegs of each colour to the opposite side of the board. A peg can be moved to an empty adjacent hole or over one peg of the opposite colour. No backward moves are allowed. What is the minimum number of moves needed to complete the game?

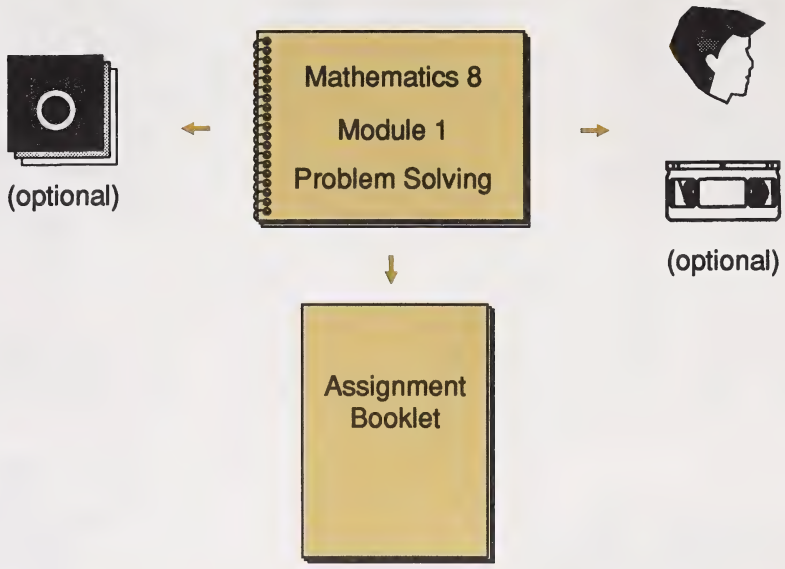


Course Overview



Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 1.

Module 1 Components



This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet. This module booklet is not to be submitted for a grade.

Your mark on this module will be determined by your work in the Assignment Booklet.

Take time to preview the module booklet before beginning Section 1.

What Lies Ahead

In this section you will learn the four stages of problem solving.

- understanding the problem
- developing a plan
- trying the plan
- looking back

Working Together

People go through different stages of development: infancy, childhood, adolescence, and adulthood. Similarly, there are different stages of problem solving. This section will deal with the four stages of problem solving.

Video Activity

Watch the video program *THINKABOUT: Find Your Guide* (Agency for Instructional Television). After you watch the video, read the notes in this section.

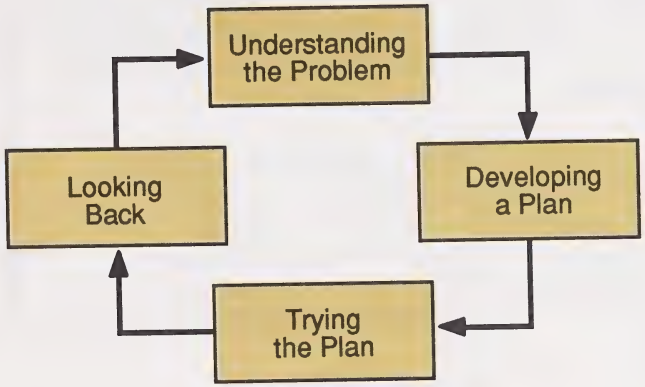
If you cannot view the video, begin reading the notes in this section now.

The Four-Stage Process

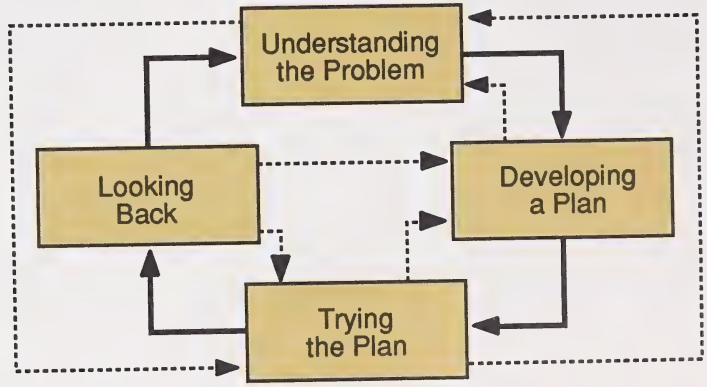
There are four stages that can be used to solve any problem.

- understanding the problem
- developing a plan
- trying the plan
- looking back

A problem can usually be approached in the order outlined in the following diagram.



If you encounter difficulties in your original approach, or if you realize that another approach will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



Each stage of the four-stage problem-solving process will now be examined in more detail.

Understanding the Problem

In this stage you should realize that you are dealing with a problem. There are various reasons for this.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Strategies for understanding the problem will be discussed in Section 2.

At this stage you should also think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer. Section 3 will discuss reasonableness of answers.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- drawing a sketch
- using objects
- making a list
- making a table
- guessing-checking-revising
- finding and applying a pattern
- simplifying the problem
- working backwards

Each of these strategies will be discussed in greater detail later in this module.

Trying the Plan

In this stage you should try the plan and see if it works.

If you are doing calculations, be sure to work carefully and record your progress (write down what you want to calculate). You are encouraged to use a calculator to help with your calculations.

Note

While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer to test it and ask yourself these questions: "So, did my plan work? Is my answer reasonable?"

If you did not arrive at an answer, or if your answer is unreasonable, you may need to repeat one or more of the earlier stages. You might have made errors while trying your plan. Perhaps another approach would work better.

Section 3 discusses reasonableness of answers. If you arrived at a reasonable answer, you may wish to consider other possible answers or other ways to solve the problem. Section 10 discusses alternative strategies to solve the same problem.

Note

People sometimes forget the looking-back stage and think they are done the problem when they find an answer. Do not overlook this important stage.

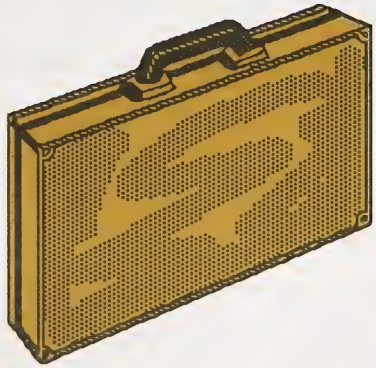
Practice Activities

Space for Your Work

1. Name the four stages in the process of problem solving.

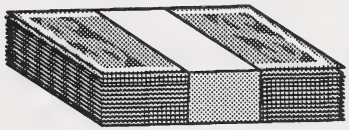
2. Explain how you would solve each of these problems.

   a. If a briefcase was packed with $10 bills, what would be the total value of the money?

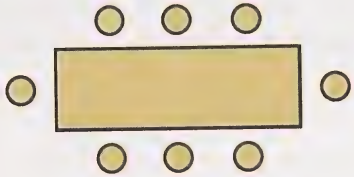


Space for Your Work

b. How would you cash a $33 cheque with six bills if you cannot use $1 bills?



c. A table in a restaurant seats eight people—three at each long side and one person at each end. How many people can be seated if four of these tables are pushed together into one long row?



3. Jackie had the following problem to solve.

   Carlos earns $12.40 each week for his work stocking grocery shelves. He also receives an allowance of $4.25 each week for helping with chores around the house. How much money does Carlos make in a year?

   Jackie calculated that Carlos earns $8500 in a year. Is her answer reasonable? Why or why not?

4. Frank had this problem to solve.

   What digit goes in the ▢, ◆, and ● to make the following equation true?

   ▢▢ × ●● = ▢◆▢

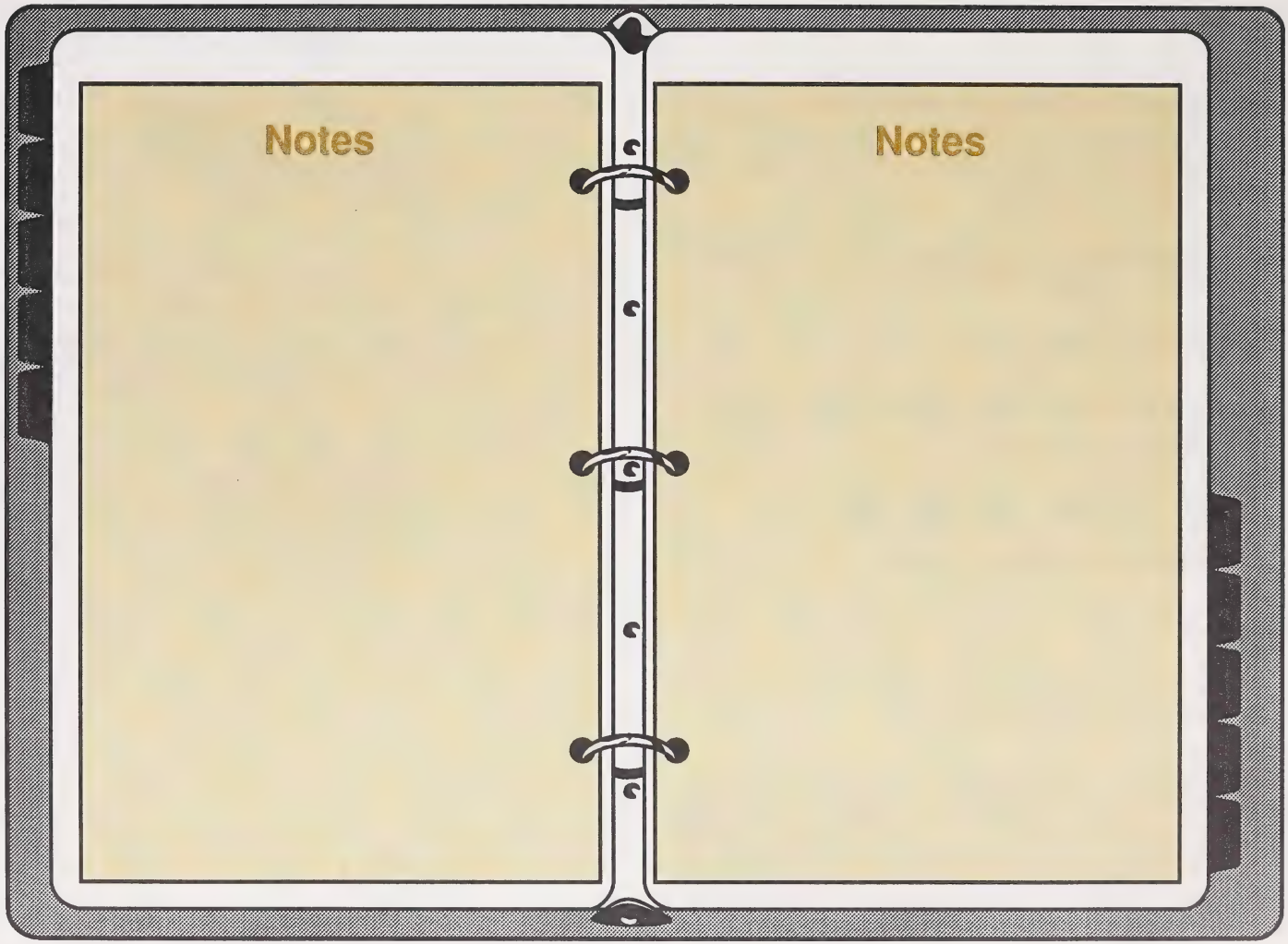
   Frank solved the problem this way.

   33 × 11 = 363

   Are there any other possible answers to the problem? If so, what are they?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.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What Lies Ahead

In this section you will learn these skills.

- identifying the essential elements of a problem (what you know and what you need to know)
- ignoring unnecessary details
- changing the setting of a problem to help understand the problem
- restating the problem in your own words

Working Together

Do you remember the four-stage problem-solving process? The first stage is understanding the problem.

This section will deal with techniques that you can use to help you understand what you are being asked to discover in a problem.

Video Activity

Watch the program *MATH WORKS: Identifying the Problem* (Agency for Instructional Television).

The program has three stories and a real-life application. Once you have watched the video, read the notes and examples in this section.

If you cannot watch the video, read the following summary of the program.

Program Summary

Story 1

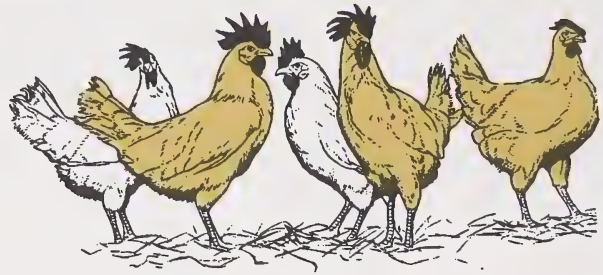
Jenny and Arlene want to give their grandmother a jewellery box for her eightieth birthday. The girls have saved some money and they want to figure out how much more they need.

Arlene thinks that "more" means that they need to use addition to solve the problem. She figures out the problem, but gets an incorrect answer. Jenny realizes that the answer is unreasonable, so they discuss what they already know and what else they need to know. Jenny includes many unneccesary details in the list of what they need to know, but Arlene points out that these should be ignored. Their discussion helps them understand that "more" actually means that they need to use subtraction to solve the problem correctly.

Story 2

Andy and Paul are doing their homework. They have been given a problem about a farmer, some hens, and some eggs. The problem seems difficult to them because they are unfamiliar with farm life.

To help them understand, they decide to change the setting of the problem. When Paul substitutes bicycles and wheels for hens and eggs, he is able to understand the new problem. This gives him ideas on how to solve the original problem.



Program Summary

Story 3

Mark and Sharon are shopping for wieners in the grocery store. They want to figure out which package of wieners to buy—a package of ten for $1.98 or a package of six for $1.08. They are confused, so they decide to restate the problem in their own words to make it more understandable.

Real-Life Application

An interior designer states that the best way to avoid problems in a detailed project is to ignore unnecessary information.

There are three techniques you can use to identify the essential elements of a problem.

- Focus on the essential information and eliminate any unnecessary information.
- Restate the problem in your own words.
- Change the setting of the problem to a more familiar one. It may give you ideas on how to solve the problem.

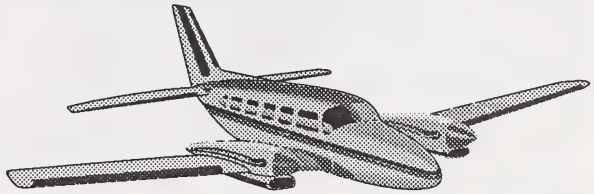
Practice Activities

Space for Your Work

1. Cross out the unnecessary details in the following problems and then restate the problems in your own words.

   a. Rock is working very hard to earn enough money to buy a new $425 bicycle. He gets paid $14 per week for babysitting two hours on Thursday and five hours on Saturday. As well, he earns $20 per week for delivering newspapers between 6:30 and 7:30 a.m. every morning. He also gets paid $5 every Wednesday for spending $1\frac{1}{2}$ hours delivering weekly advertising flyers. For how many weeks will Rock have to work so that he can buy his new bicycle?

   b. A plane leaves Vancouver at 7:30 a.m. and arrives in Toronto at 12:00 noon Vancouver time. It flies a distance of 3400 km at a cruising speed of 950 km/h. When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?



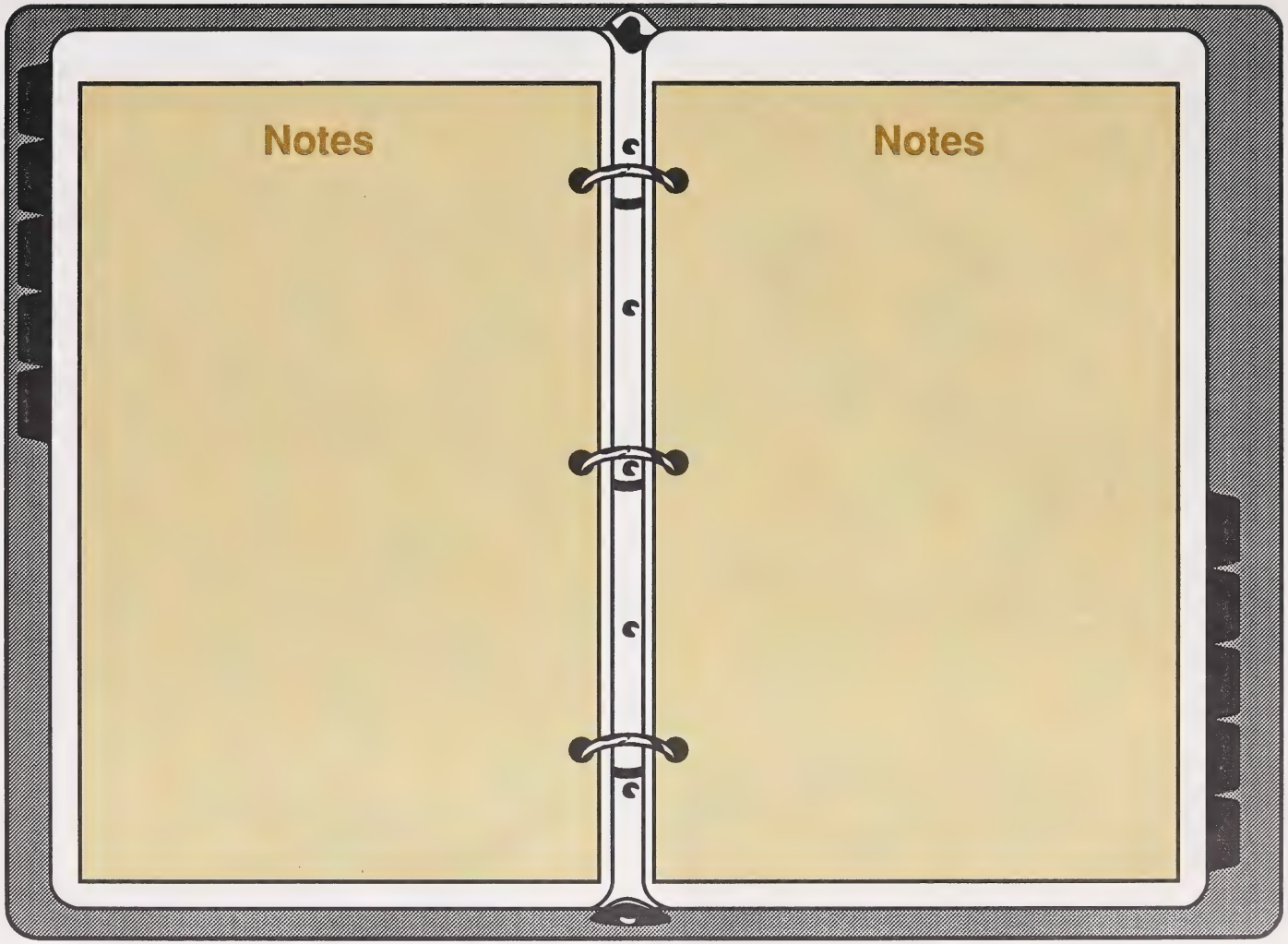
2. Change the setting of the following problems.

Space for Your Work

a. A numismatist bought a rare coin for $30, sold it for $40, bought it back for $60, and sold it again for $70. How much money did the numismatist make or lose in buying and selling this stamp?

b. Armond spent twice as much on shantung as he spent on chambray. If he spent $30 on chambray, how much did he spend on shantung?

See your learning facilitator to check your answers and to receive further instructions.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What Lies Ahead

In this section you will learn these skills.

- estimating an answer
- determining if an answer is reasonable

Working Together

When problem solving, it is important to spend time estimating the answer and looking back to see if the answer is reasonable.

Video Activity

Watch the video *SOLVE IT: Reasonableness of Answers* (Agency for Instructional Television). The program has three stories and a real-life situation. As you watch the program, look for situations in which the characters need to check the reasonableness of their answers. Be sure to notice the methods they use to do this.

If you cannot watch the video, read the following summary of the program.

Program Summary

Story 1

A teacher poses two math problems to the class. Toby answers incorrectly because he does not estimate the answer and he does not think about the reasonableness of his answer.

Ebony, on the other hand, first makes a mental picture and thinks about the problem situation in order to imagine what the answer will be like. Afterwards, she thinks about the reasonableness of her answer by comparing her answer to her estimate.

Story 2

Lisa and Bobby are fencing their front yard. They must fence 6 m of the yard, and the fence comes in sections 2 m wide. Bobby figures out the problem incorrectly because he does not think about the problem situation and he overlooks thinking about the reasonableness of his answer.

Lisa, on the other hand, realizes that Bobby's answer is unreasonable because she has made a mental picture of the situation.

Story 3

Toby and Gina are helping the owner of a sporting goods store. Their job is to find the total value of the bicycles in the store. Toby makes an estimate of the cost of the bicycles before he begins calculating. This helps him check the reasonableness of his answer afterwards.

Real-Life Situation

A pediatrician (children's doctor) explains how she uses problem solving in her job. She uses the same four-stage process as you. First she tries to understand the problem. Then she makes a diagnosis, prescribes a treatment, and checks the results. If the treatment is effective, she has solved the problem. If the treatment is not effective, she must think about the problem again and plan another treatment.

Practice Activities

Space for Your Work

For Questions 1 to 3, estimate the answers and then tell which of the provided answers is the most reasonable and explain why.

1. If a person walks 1 km, how many steps are taken?

   a. 25
   b. 100
   c. 2000

2. A hamburger patty has a mass of about 100 g. How much ground beef is needed if 25 people eat 2 hamburgers each?

   a. 5 kg
   b. 50 kg
   c. 500 kg

3. If you use 2 mL of toothpaste each time you brush your teeth (3 times a day), about how many weeks will it take to use up a 100-mL tube of toothpaste?

   a. 2 weeks
   b. 6 weeks
   c. 12 weeks

For Questions 4 to 6, calculate the answers and then tell which of the provided answers is the most reasonable and explain why.

Space for Your Work

4. The 29 students in the seventh-grade class at the Willow Creek School are going on a field trip. If four students fit in each car, how many cars are needed?

   a. 7
   b. 1
   c. 7.25
   d. 8

5. Charlie's jazz band rehearsed a total of 29 hours in the last 4 days before the concert. If they spaced their rehearsals equally over the 4 days, how many hours did they practice each day?

   a. 7
   b. 1
   c. 7.25
   d. 8

6. Annette has collected 29 cassette tapes. She wants to arrange them in a box which will hold 4 tapes in each row. How many tapes will she put in the last row?

   a. 7
   b. 1
   c. 7.25
   d. 8

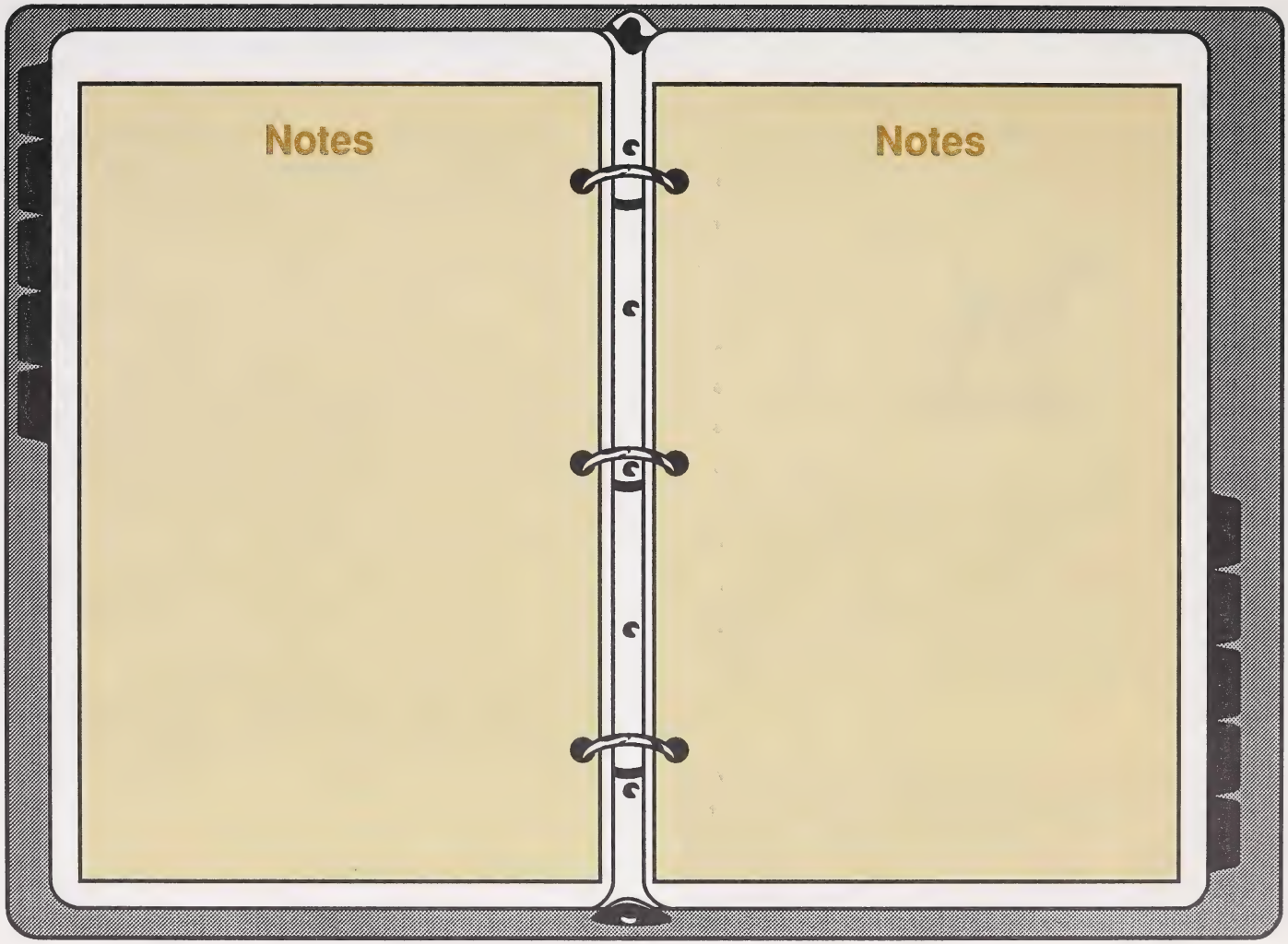
For Questions 7 and 8, draw a picture to help you get a reasonable answer.

Space for Your Work

7. Nadine is building a dog run. She wants the run to be a rectangle with 6 upright posts on each short side and 10 upright posts on each long side. How many posts will she need?

8. Gail's office is 8 km from her home. Yesterday morning she drove 2 km before she realized she had forgotten her briefcase. She returned home to get her briefcase and then drove to her office. At noon Gail drove 4 km to run some errands. She then went back to the office for the rest of the afternoon. At the end of the day she drove straight home. How far did she drive yesterday?

See your learning facilitator to check your answers and to receive further instructions.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What Lies Ahead

In this section you will learn these skills.

- using sketches to solve problems
- using objects to solve problems

Working Together

There are many strategies you can use to help you understand a problem. This section deals with using objects and drawing sketches to help solve problems.

Example 1

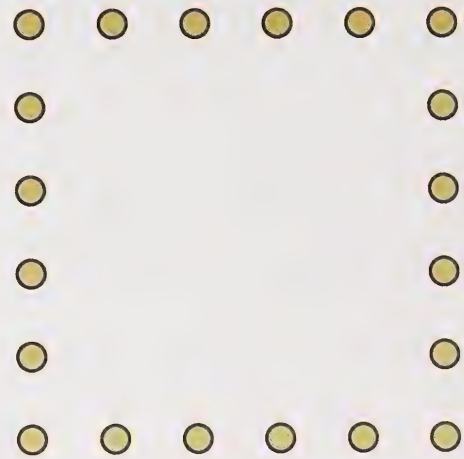
Twenty posts are used to enclose a square lot. If the posts are placed 3 m apart, what is the length of one side of the lot?

Solution

You could use objects to represent the posts, or you could make a sketch.

The advantage of using objects to represent the posts is that you can easily rearrange them.

It does not matter if you use objects or if you make a sketch—the result will be the same. You will find that the posts must be arranged as shown in the following diagram.



From the diagram you can see that there are 5 spaces between the posts on one side of the square. If the posts are placed 3 m apart, the length of one side of the lot can easily be calculated.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

Example 2

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

You could use paper cups to represent the days. The cups could be labelled as shown below.

Begin at the cup representing the day after tomorrow and count back four days.

4	3	2	1	
Monday	Tuesday	Wednesday	Thursday	Friday

You will find that Monday was the day before yesterday. You will also discover that today is Wednesday.

You could make a sketch instead of using objects.

4	(Monday)	the day before yesterday
3	(Tuesday)	yesterday
2	(Wednesday)	today
1	(Thursday)	tomorrow
	(Friday)	the day after tomorrow

Today is Wednesday.

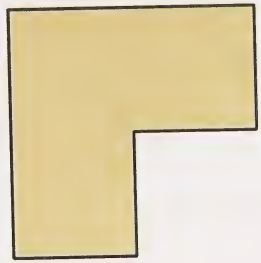
Practice Activities

Space for Your Work

1. Draw sketches to help you solve the following problems.

   a. A group of cross-country skiers intend to camp overnight at a site 15 km directly north of the parking lot. Starting at 8:00 a.m., they ski 6 km north, 4 km west, 12 km north, 7 km east, and 3 km south before stopping for lunch at noon. In what direction and how far must they ski after lunch to reach the campsite?

   b. A developer has bought this L-shaped piece of land. The developer wants to subdivide it into three congruent lots (lots that have the same shape and size). How can this be done?



2. Use objects to help you solve the following problems.

Space for Your Work

a. Assuming the length of a side of a triangle equals one toothpick, use six toothpicks to make four identical triangles.

b. How would you fold this sheet of paper so that the numbered sections lie on top of one another in order from 1 to 8?

3	4	2	7
6	5	1	8

Computer Alternative

Space for Your Work

3. Museum Trip

You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

From the Chapter Menu choose "5 Using Models." Then choose "1 Museum Trip."

After you've read the problem, decide how much help you want.

Follow the directions on the screen.

Note: Whenever a smiling face appears, the computer stops. To make it continue press RETURN.

4. Measuring Puzzle

You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

From the Chapter Menu choose "5 Using Models." Then choose "Measuring Puzzle."

Complete as many Measuring Puzzles as you wish at the computer.

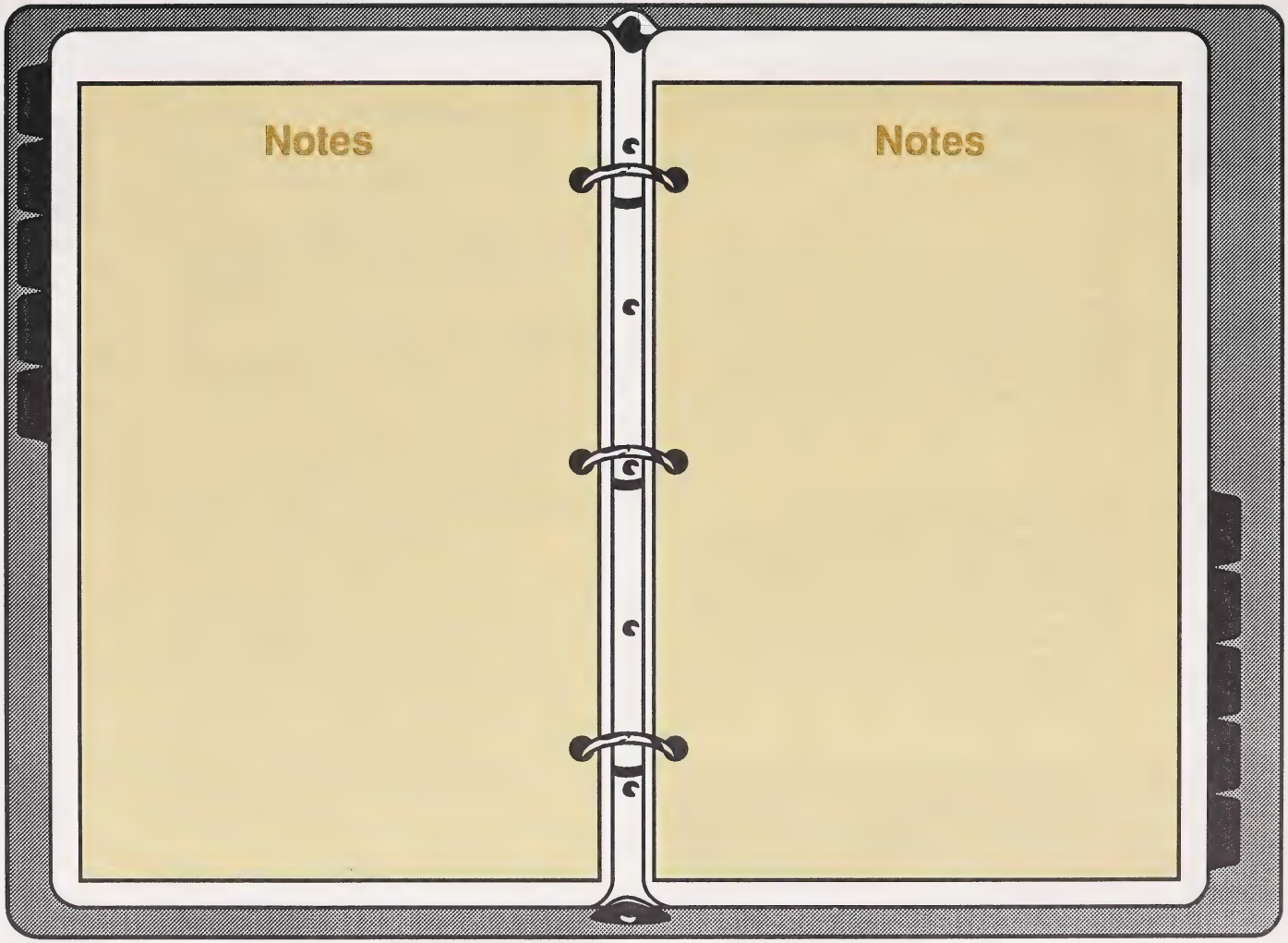
Space for Your Work

5. How can you cover half of a 1 m square window and still have a square opening that is 1 m across and 1 m from top to bottom?

6. A spider is at the bottom of a 8 m deep well. The spider climbs up 4 m each day, but slips back down 3 m each night. How many days will it take the spider to reach the top of the well?

7. Sandy wants to make a square breadboard from a rectangular piece of wood that measures 24 cm by 54 cm. She wants to use all the wood. What is the length of the side of the largest breadboard that Sandy can make?

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

In this section you will learn these skills.

- making lists to solve problems
- making tables to solve problems

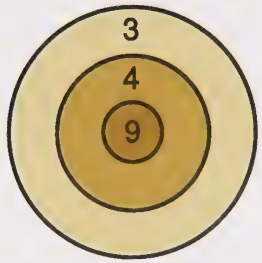
Working Together

Some problems have a lot of data (facts and information).

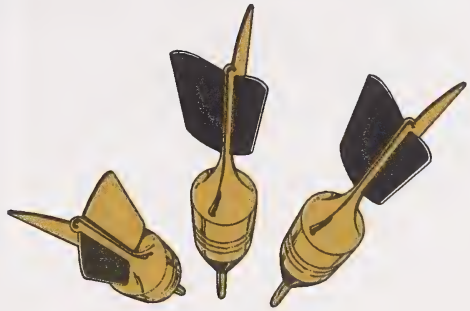
This section deals with two ways that you can organize the data to help solve a problem: making a complete organized list, and making a table.

Example 1

John and his friends like playing a game called *Bull's Eye* with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.



If John throws three darts and all three hit the target, how many different point totals are possible?



Solution

You can make a complete list of all the possibilities. The list arranges the point totals from the highest possible score to lowest possible score.

$9 + 9 + 9 = 27$

$9 + 9 + 4 = 22$

$9 + 9 + 3 = 21$

$9 + 4 + 4 = 17$

$9 + 4 + 3 = 16$

$9 + 3 + 3 = 15$

$4 + 4 + 4 = 12$

$4 + 4 + 3 = 11$

$4 + 3 + 3 = 10$

$3 + 3 + 3 = 9$

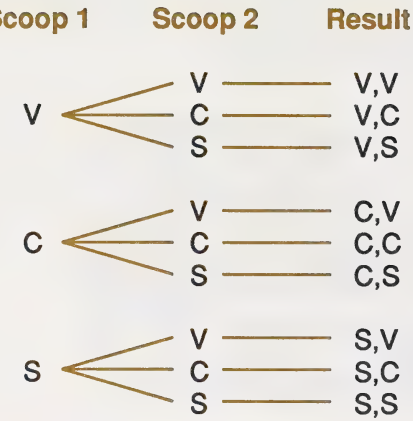
There are ten possible point totals.

Example 2

At an ice cream parlour there are three flavours of ice cream. How many different-looking double-scoop ice-cream cones are possible? Hint: A chocolate-vanilla cone is different from a vanilla-chocolate cone. Order makes a difference.

Solution

You can draw a **tree diagram** to list the combinations. Use V for vanilla, C for chocolate, and S for strawberry.



Nine different double-scoop ice-cream cones are possible.

Example 3

There are 8 vehicles in a parking lot. Some are motor bikes and some are cars. Sam counted 26 wheels altogether. How many of the vehicles are motor bikes and how many are cars? Hint: Motor bikes have 2 wheels and cars have 4 wheels.

Solution

You can arrange the data in a table.

Number of motor bikes	1	2	3	4	5	6	7
Number of cars	7	6	5	4	3	2	1
Number of wheels	30	28	26	24	22	20	18

The table shows that there are 3 motor bikes and 5 cars in the parking lot.

Practice Activities

Space for Your Work

1. Kenza counted 29 wheels in a children's parade of bicycles and tricycles. If there were 12 children in the parade, how many tricycles were there?

2. Using only quarters, dimes, and nickels, how many ways can Philip pay for 95 cents worth of jelly beans?

3. When choosing a new car at Magnificent Motors, the buyer may choose from 3 different models, 7 different colours, and either automatic or standard transmission. How many different car combinations are available?

4. Kimberley does housecleaning for her grandmother every Saturday. She earns 15 cents for each room that she vacuums and 25 cents for each room that she dusts. One day she did 16 jobs and was paid $3.30. How many rooms did she dust?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- solving problems by guessing, checking, and revising your answer
- organizing your guesses

Working Together

This section deals with another useful strategy called guessing-checking-revising.

This strategy involves using what you know about the situation to make a reasonable guess, checking the guess against the conditions in the problem, and using the findings of the test to revise the guess.

Video Activity

Watch *SOLVE IT: Guess-Check-Revise* (Agency for Instructional Television) to discover how the guessing-checking-revising strategy can be used to solve problems. Then read the notes on using a table to organize your guesses.

If you cannot view the video, read the following summary of the program. (The program has a story and a real-life application.) Also read the notes on using a table to organize your guesses.

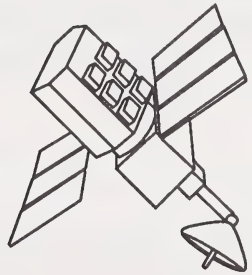
Program Summary

Story

Claude and Keisa's family has moved into a new apartment and they have two problems. They need to move a large chair through a narrow doorway and they need to hook up the cables to the VCR. Claude and Keisa solve these problems through a process of guessing, encountering difficulty, making better guesses on the basis of what they have learned, and guessing again.

Real-Life Application

A project planner explains how the guess-and-test strategy is an essential part of planning the Voyager space missions.



Using a Table to Organize Guesses

It is often helpful to make a table to help you arrange your guesses.

Study the following example. The solution is on the next page.

Example

Two boys are talking about their baseball cards.

Matt said to Jon, "Give me one of your cards and I will have as many as you have."

Jon said to Matt, "Give me one of your cards and I will have twice as many as you have."

How many cards does each boy have?

Solution

To help you understand the problem, you could use objects to represent the cards and act out the problem. You should realize that the numbers are whole numbers.

Use the guess and test strategy. First make a reasonable guess and then act out the problem using the objects.

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.
- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Record your guess in a table. Test your guess. If your guess does not fit the conditions, revise your guess and repeat the process.

	Matt's Cards	Jon's Cards	Tests	
			First Condition	Second Condition
Guess 1	1	4	$1 + 1 = 2,\ 4 - 1 = 3;\ 2 \neq 3$	$1 - 1 = 0,\ 4 + 1 = 5;\ 5 \neq 2 \times 0$
Guess 2	2	4	$2 + 1 = 3,\ 4 - 1 = 3;\ 3 = 3$	$2 - 1 = 1,\ 4 + 1 = 5;\ 5 \neq 2 \times 1$
Guess 3	3	5	$3 + 1 = 4,\ 5 - 1 = 4;\ 4 = 4$	$3 - 1 = 2,\ 5 + 1 = 6;\ 6 \neq 2 \times 2$
Guess 4	4	6	$4 + 1 = 5,\ 6 - 1 = 5;\ 5 = 5$	$4 - 1 = 3,\ 6 + 1 = 7;\ 7 \neq 2 \times 3$
Guess 5	5	7	$5 + 1 = 6,\ 7 - 1 = 6;\ 6 = 6$	$5 - 1 = 4,\ 7 + 1 = 8;\ 8 = 2 \times 4$

Guess 5 satisfies both conditions.

Matt has 5 cards and Jon has 7 cards.

Practice Activities

Space for Your Work

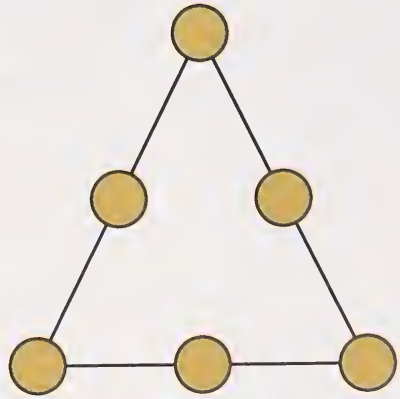
Computer Alternative

1. Do the program "Guess and Test" on Disk A of *MAC 7* (Houghton-Mifflin).

Print Alternative

2. Carl opened his textbook and noticed that the product of the numbers of the two open pages was 2756. To which pages was the book opened?

3. Place the numbers 2, 3, 4, 20, 30, and 40 in the circles so that the product of the three numbers on each side of the triangle is 240.



4. A physical education instructor spent $231 for 16 new balls. If softballs costs $4 each, footballs cost $24 each, and soccer balls cost $33 each, how many of each kind did the instructor buy?

Space for Your Work

Guess	Number of Softballs	Number of Footballs	Number of Soccer Balls	Test
1	3	4	9	3 × $ 4 = 12 4 × $24 = 96 9 × $33 = 297 Total 405
2				
3				
4				

5. The sum of the present ages of two sisters is 24. Six years ago Chris was twice as old as Tina. What are their present ages?

Space for Your Work

Guess	Chris' Age Six Years Ago	Tina's Age Six Years Ago	Test
1	6	3	$6 + 6 = 12$ $3 + 6 = \underline{\ 9}$ 21
2			
3			
4			

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- making a simpler model of a complex problem in order to find a pattern
- applying the pattern to solve the problem

Working Together

Have you ever been confused because a problem is too complex? This section deals with one way to simplify a problem: finding and applying a pattern.

Video Activity

Watch Story 1 of *MATH WORKS: Problem Solving: Simplifying the Problem* (Agency for Instructional Television) and Story 2 of *SOLVE IT: Solving a Simpler Problem* (Agency for Instructional Television).

If you cannot view the videos, read the following summary of the programs.

Program Summary

***MATH WORKS: Problem Solving: Simplifying the Problem*—Story 1**

There are 50 people at a party. Each person wants to shake hands with everyone else. The host points out that it will be a complex problem to find out exactly how many handshakes will be exchanged.

The host demonstrates that the problem can be simplified by using a simpler model. When a smaller number of people is used, a pattern can be discovered.

The pattern can be used to solve the problem with 50 people.

The problem was acted out with 6 people and the results were recorded.

Below is the table from the program.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4
6	15	+5

+1, +1, +1, +1

Once you know the pattern, you can calculate how many handshakes will be exchanged by 50 people. Use a calculator.

The answer is 1225 handshakes.

Program Summary

SOLVE IT: Solving a Simpler Problem—Story 2

Tree, Sam, and Mike must figure out a bowling schedule for 20 students. Each student must bowl against every other student. They decide to use a simpler model to calculate how many games are needed.

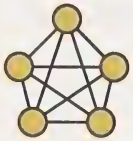
First they consider a schedule for three students. They make a diagram of this. The circles represent the students. The lines represent the games needed.

Three games are needed.

Then they consider a schedule for four students.

Six games are needed.

Then they consider a schedule for five students.



Ten games are needed.

Next they look for a pattern that they can use to solve the problem involving 20 students.

Number of students	1	2	3	4	5
Games needed	0	1	3	6	10

Pattern

+1 +2 +3 +4

+1 +1 +1

Next they applied the pattern. Six students require 10 + 5, or 15, games. Seven students require 15 + 6, or 21, games.

You can use the pattern to find how many games are needed by 20 students. You can use a calculator to solve the problem.

The answer is 90 games.

Practice Activities

Computer Alternative

1. Calculator Practice

The computer program you will be using has a built-in calculator to help you do computations. The following exercise will show you how to use this built-in calculator.

You will be working at the computer, so take this booklet and Disk 1 of the *MATH STRATEGIES: Problem Solving* (SRA) with you. Run the disk and wait until you see the Chapter Menu on the screen.

Type "1." Press RETURN. Follow the directions on the screen until you see the following chart.

KEY:

Plus	+
Minus	–
Times	× or *
Divided by	/
Is equal to	RETURN

Notice that you may type either [×] or [*] for multiplication. Also, since there is no [÷] key, use the [/] key for division instead. You need to use the SHIFT key when typing [+] or [*] .

Type "46 + 38." Press RETURN. Here's what should be on the screen:

46 + 38 = 84

On the next line, you'll see a ":" followed by a flashing "—." This is the signal that the calculator is ready for the next calculation.

Try holding the SHIFT key and press the [=] key. As you can see, the "=" sign is not used on this calculator. Whenever you're ready for the calculator to show the answer, press RETURN instead.

Type "876." Then press the [←] key once. The 6 is erased. When you want to erase something you have typed, use the [←] key. (Note: Once you have pressed RETURN, you cannot erase what you have typed.)

To erase all the work that is on the screen, type "E". Try it.

If you want to type a 5-digit number, say 34 672, you will have to type "34672". The calculator will not accept spaces.

2. Spy Ring

Choose "Program 5 Using Models" from Disk 1 of the *MATH STRATEGIES: Solving Problems* (SRA) package. Then do the "Spy Ring Problem."

Note

When you see ":—" on the screen it means that you can use the built-in calculator to do calculations. After you have done the calculations, you must enter your answer and press RETURN.

Print Alternative

For Questions 3 to 5, you may act out the problems or draw diagrams in order to help you find the patterns. Apply the patterns to solve the problems.

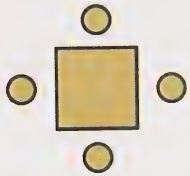
3. There are eleven students attending a summer camp. If two students were introduced to each other at a time, how many introductions were made?

Space for Your Work

4. What is the largest number of sections into which a circle can be divided with ten straight lines?

Space for Your Work

5. One person can be seated on each of the four sides of a square table. If 14 square tables are arranged in one long row, how many people can be seated?



See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- using smaller numbers to solve a problem with large numbers
- breaking problems into steps

Working Together

This section deals with two ways to simplify problems:

- using a simpler model with smaller numbers when the numbers in a problem seem too large to work easily
- breaking the problem into steps when the problem seems too complicated

Video Activity

Watch Stories 1 and 3 of *SOLVE IT: Solving a Simpler Problem* (Agency for Instructional Television), and Stories 2 and 3 of *MATH WORKS: Simplifying the Problem* (Agency for Instructional Television) to see how each kind of simplification strategy is used.

If you cannot view the videos, read the following summaries of the programs.

Program Summary

SOLVE IT: Solving a Simpler Problem—Story 1

Tree, Sam, and Mike want to figure out the average distance from Earth to Mars.

They know that the average distance from the sun to Mars is 228 000 000 km and the average distance from the sun to Earth is 150 000 000 km. They are overwhelmed by the large numbers and cannot figure out the problem, so they decide to simplify it.

They construct a simpler model which they can solve.

Sam and Tree both live east of the store. Sam lives 3 km from the store and Tree lives 6 km from the store. How far is it from Sam's house to Tree's house?

The distance between Sam's house and Tree's house is found by subtracting.

6 km – 3 km = 3 km

It is 3 km from Sam's house to Tree's house.

Now it is easier to solve the original problem.

The simpler model gives Tree, Sam, and Mike an idea for solving the original problem. They use objects to represent the sun, Earth, and Mars.

Now they understand that the distance between Earth and Mars can be found by subtraction.

228 million km – 150 million km = 78 million km

The distance from Earth to Mars is 78 000 000 km.

SOLVE IT: Solving a Simpler Problem—Story 3

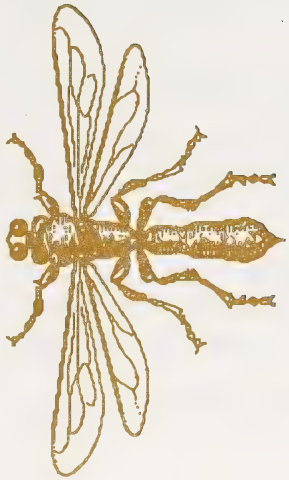
Tree, Sam, and Mike find a coded note and they try to figure out its meaning. They are confused because a number of steps are involved. They decide to decode one piece of the note at a time. By doing this they succeed in understanding the note.

Program Summary

MATH WORKS: Simplifying the Problem—Story 2

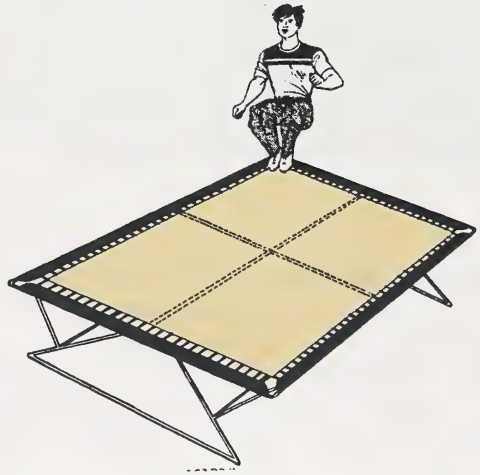
Carlos wants to write a report about all the insects that live in his state. When Carlos discovers that there are 88 600 insect species in North America, he thinks the problem is too big to handle.

The librarian suggests that Carlos should break down the problem into smaller pieces and tackle the pieces step by step.



MATH WORKS: Simplifying the Problem—Story 3

Carlos, Sonia, and Won-Li have volunteered to help the gymnastics coach raise money for a trampoline.



They plan to sell candy, but first they need to find out how much to charge per box. They find the problem difficult until they substitute smaller numbers. By solving a problem with smaller numbers, they discover how to solve the original problem.

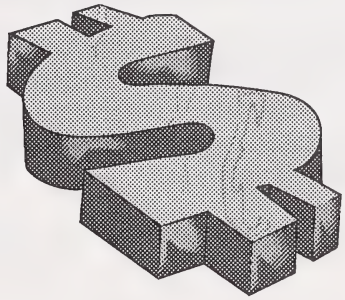
Practice Activities

Space for Your Work

For Questions 1 and 2, use simpler problems to help you solve the original problems. You can change the settings and the numbers, but do not change the operations.

1. The total Canadian national debt in 1977 was $29 585 900 000. In 1987 the national debt was $264 101 000 000. During this period of time, how much did Canada's national debt increase?

2. On December 31, 1986 there were 1 157 021 000 bills of Canadian paper money in circulation. Of these, 335 727 000 were one-dollar bills and 335 173 000 were twenty-dollar bills. How many bills were there of each of the other denominations (two-, five-, ten-, fifty-, hundred-, and thousand-dollar bills)?



Computer Alternative

3. Do "Lightning Strikes" from the *MATH STRATEGIES: Solving Problems* (SRA) package.

   You will need Disk 1. Start Disk 1. When the Chapter Menu comes on the screen, type "2" and press RETURN. You will then see the menu for Simplifying Problems.

   Type "1" for "Lightning Strikes" and press RETURN.

   Follow the instructions on the screen.

4. Do "A Beautiful Dream" from *MATH STRATEGIES: Solving Problems* (SRA).

   Use Disk 2. From the Chapter Menu, choose "3 Problem Breakdown." Then, choose "2 A Beautiful Dream."

   Solve the problem at the computer.

Space for Your Work

Print Alternative

Space for Your Work

In Questions 4 and 5 simplify the problems by doing them in steps.

5. The students in a Grade 8 class have a total of 51 pets. Five students have one pet each, eight students have three pets each, three students have none, and the rest have two pets each. How many students have exactly two pets?

6. Rohan charges $10 per hour to clear the neighbours' driveways. His father charges him $2 per hour for use of the snowblower. How much profit does Rohan make if he and a friend work 27 hours and they share the profits equally?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- working backwards to solve problems
- using flow charts

Working Together

Usually in mathematics, you are given information and then asked to find the answer. Sometimes, however, you are given the answer and then you are asked to find a piece of information. In cases like these you need to work backwards.

Example 1

Cayley wants to stack 36 cans in the shape of a triangle. Each row must have one less can than the row below it, and the top row can only have one can. How many cans must Cayley put in the bottom row?

Solution

Normally you would start at the bottom of the triangle and work upwards like this.

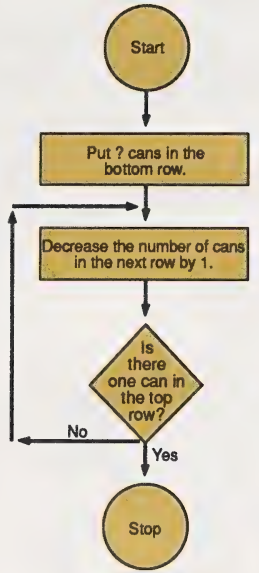
Cayley does not know how many cans to put on the bottom row, but she does know that she needs to stack 36 cans.

To solve the problem Cayley can work backwards. She can start at the top and work downwards until all 36 cans are stacked.

Cayley finds that she must put 8 cans in the bottom row.

Sometimes flow charts are helpful when solving problems that require you to work forwards or backwards. A **flow chart** is a diagram that shows a sequence of events or instructions.

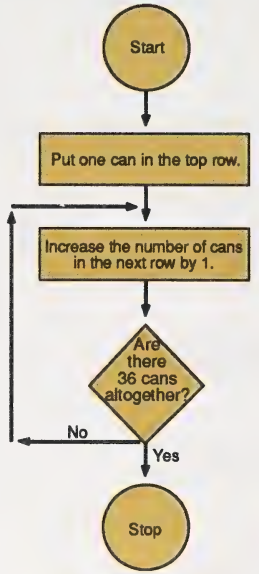
If Cayley worked forwards, the flow chart would look like this.



There are 36 cans altogether.

Since Cayley did not know how many cans to put in the bottom row she could not work forwards, but she could work backwards.

There are 36 cans altogether.



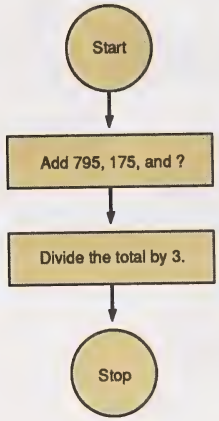
There are 8 cans in the bottom row.

Example 2

Jason plays in a bowling league. His average in the league is 195. Today Jason scored 195 and 175 in his first two games. What minimum score must he make in his third game to maintain his average.

Solution

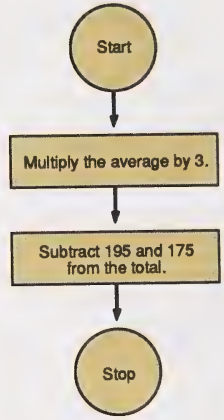
First consider how an average problem would be done if you worked forward.

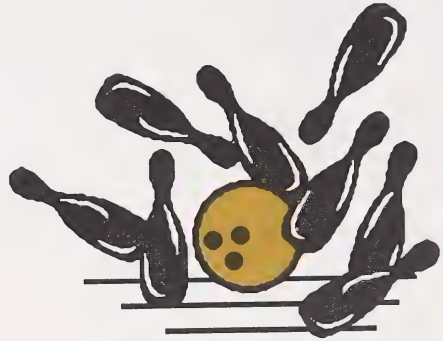


The average is 195.

You do not know one of the three scores, so you cannot do the first step, but you can work backwards.

You know that the average is 195.





You can find the third score by doing the following steps.

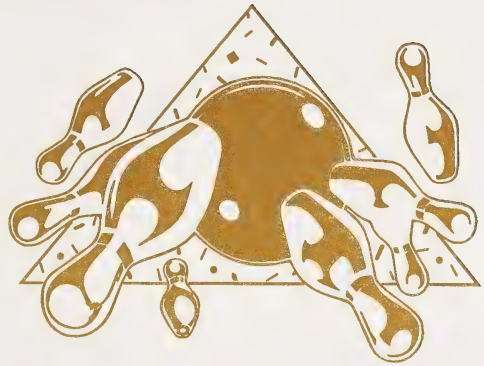
Step 1: Multiply the average by 3.

$$\begin{array}{r} 195 \\ \times\ 3 \\ \hline 585 \end{array}$$

Step 2: Subtract 195 and 175 from the total.

$$\begin{aligned} & 585 - (195 + 175) \\ = \ & 585 - 370 \\ = \ & 215 \end{aligned}$$

The third score is 215.



You can check to see if 215 is the correct third score by working forward.

Step 3: Add the three scores.

$$\begin{array}{r} 215 \\ 195 \\ +\ 175 \\ \hline 585 \end{array}$$

Step 4: Divide the total by 3.

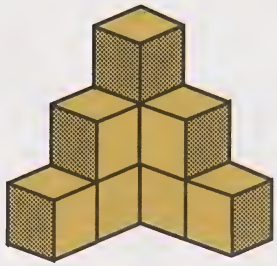
$$\begin{array}{r} 195 \\ 3\overline{)585} \\ \underline{3} \\ 28 \\ \underline{27} \\ 15 \\ 15 \end{array}$$

The average is 195, so you know that Jason must get 215 in his last game.

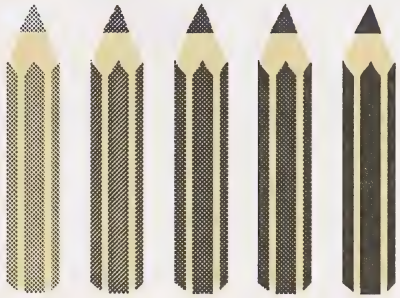
Practice Activities

Space for Your Work

1. Boxes can be stacked in a corner so that each layer has two less boxes than the layer below it, and the top layer has one box. How many boxes would be in the bottom layer if 36 boxes were stacked this way?



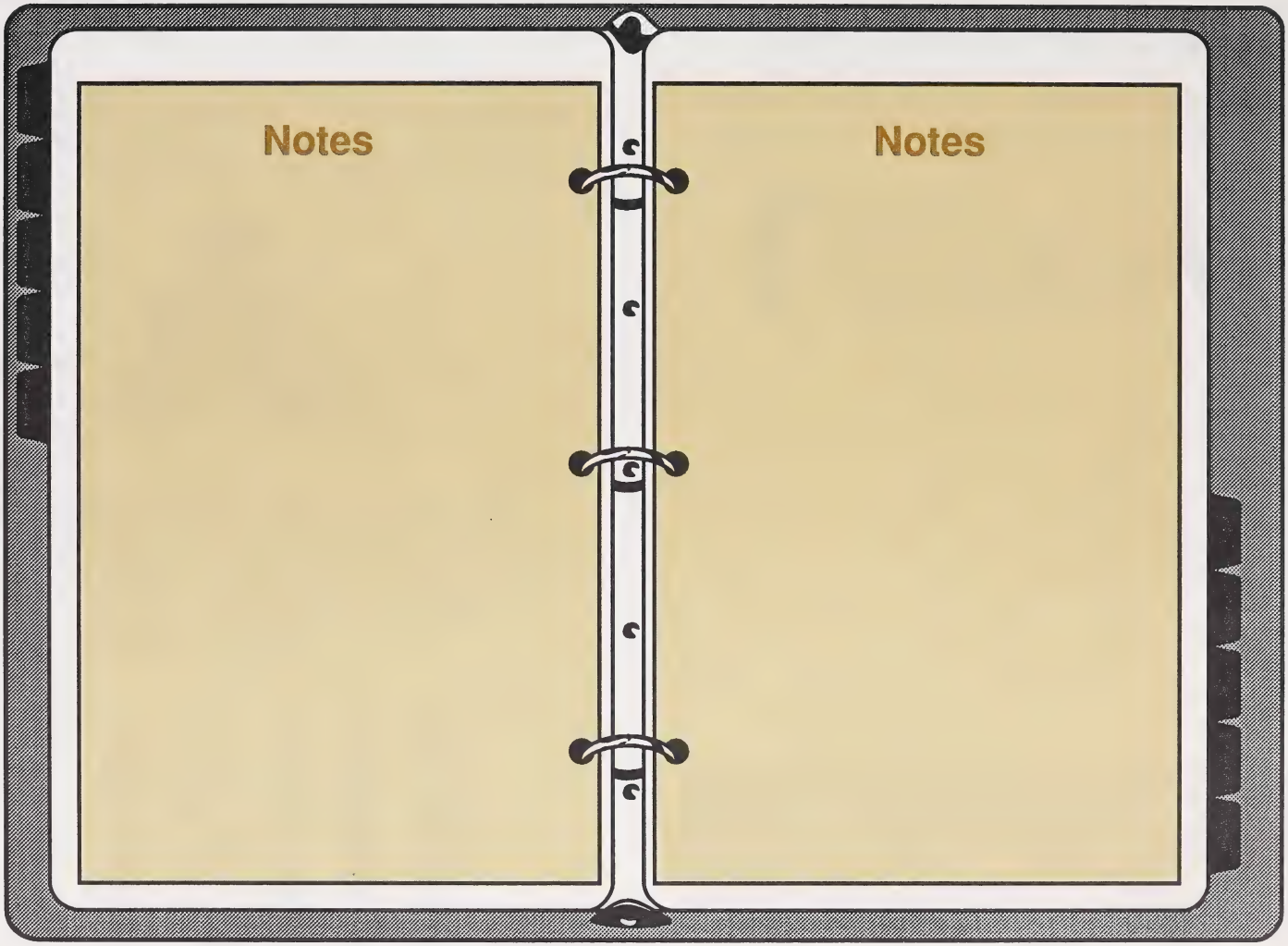
2. At the end of the school day, a teacher had 15 pencils left in her loan box. She remembered giving out 17 pencils in the morning and getting 11 back before noon, and then giving out 13 more pencils after lunch hour. How many pencils did she have at the beginning of the day?



3. Curly gave Larry and Moe as much money as each had. Then Larry gave Curly and Moe as much money as each had. Then Moe gave Curly and Larry as much money as each had. If each one now has $8, how much did each have at the beginning?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn that there are many ways to solve one problem.

Working Together

In this module you have been learning strategies to solve problems. You have probably discovered that there are many ways to solve a particular problem. This section stresses this idea.

Video Activity

Watch the program *THINKABOUT: There are Many Ways to Go* (Agency for Instructional Television). Then study the example.

If you cannot view the video, read the following program summary.

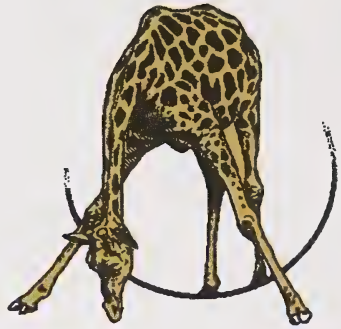
Program Summary

Kathleen, Leah, and Heather are young ice skaters preparing for a figure-skating competition.

They each tackle the problem of preparing for the competition in a different way.

Kathleen decides to do research. She goes to the library and reads old newspaper reports on the competition. Then she interviews skaters who have competed in the competition in the past. After she finds out what the judges will be looking for, she plots her movements on a drawing of the rink.

Heather decides to observe animals at the zoo to get inspiration for graceful movements. She uses her creativity by choosing music to help convey the feelings she is trying to express.



Leah experiments. Her brother videotapes her practice sessions to help her plan her routine.

Mathematical problems can also be done in several ways.

Example 1

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there?
Hint: Chickens have 2 legs and sheep have 4 legs.

Solution

Method 1

You could guess, check, and revise.

	Guess 1	Guess 2	Guess 3
Chickens	2	4	6
Sheep	8	6	4
Legs	36	32	28

Guess 3 satisfies both conditions.

There are 6 chickens and 4 sheep.

Method 2

You could make a diagram.

There are 10 animals and every animal has at least 2 legs.

This group of animals had 28 legs, so you need to add 8 legs to the drawing. Add the legs in pairs to each animal picture.

There are 4 sheep and 6 chickens.

Example 2

How many chords can be drawn joining five points on a circle? Hint: A chord joins any two points on the circumference of a circle.

Solution

Method 1

You could use simpler models and look for a pattern.

Number of Points		Number of Chords
	1	0
	2	1
	3	3
	4	6

Pattern

+1

+2

+3

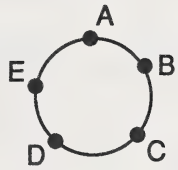
+1

+1

You can apply this pattern. A circle with five points has ten chords.

Method 2

You could make a complete list of chords. Start by labelling the points from A to E.



Then list the chords from each point.

The chords from A:
AB
AC
AD
AE

The chords from B:
BC
BD
BE
~~BA~~ ← This has already been mentioned.

The chords from C:
CD
CE
~~CA~~
~~CB~~
} These have already been mentioned.

The chords from D:
DE
~~DA~~
~~DB~~
~~DC~~
} These have already been mentioned.

The chords from E:
~~EA~~
~~EB~~
~~EC~~
~~ED~~
} These have already been mentioned.

Count how many chords there are.

There are ten chords joining five points.

Practice Activities

Space for Your Work

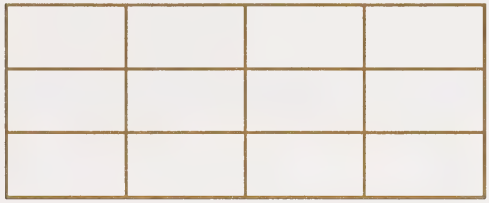
Computer Alternative

1. Do the programs "Diagonals" and "Squares" in the *MECC: Problem Solving Strategies* package.

Print Alternative

For Questions 2 and 3 use two different methods to solve the problems.

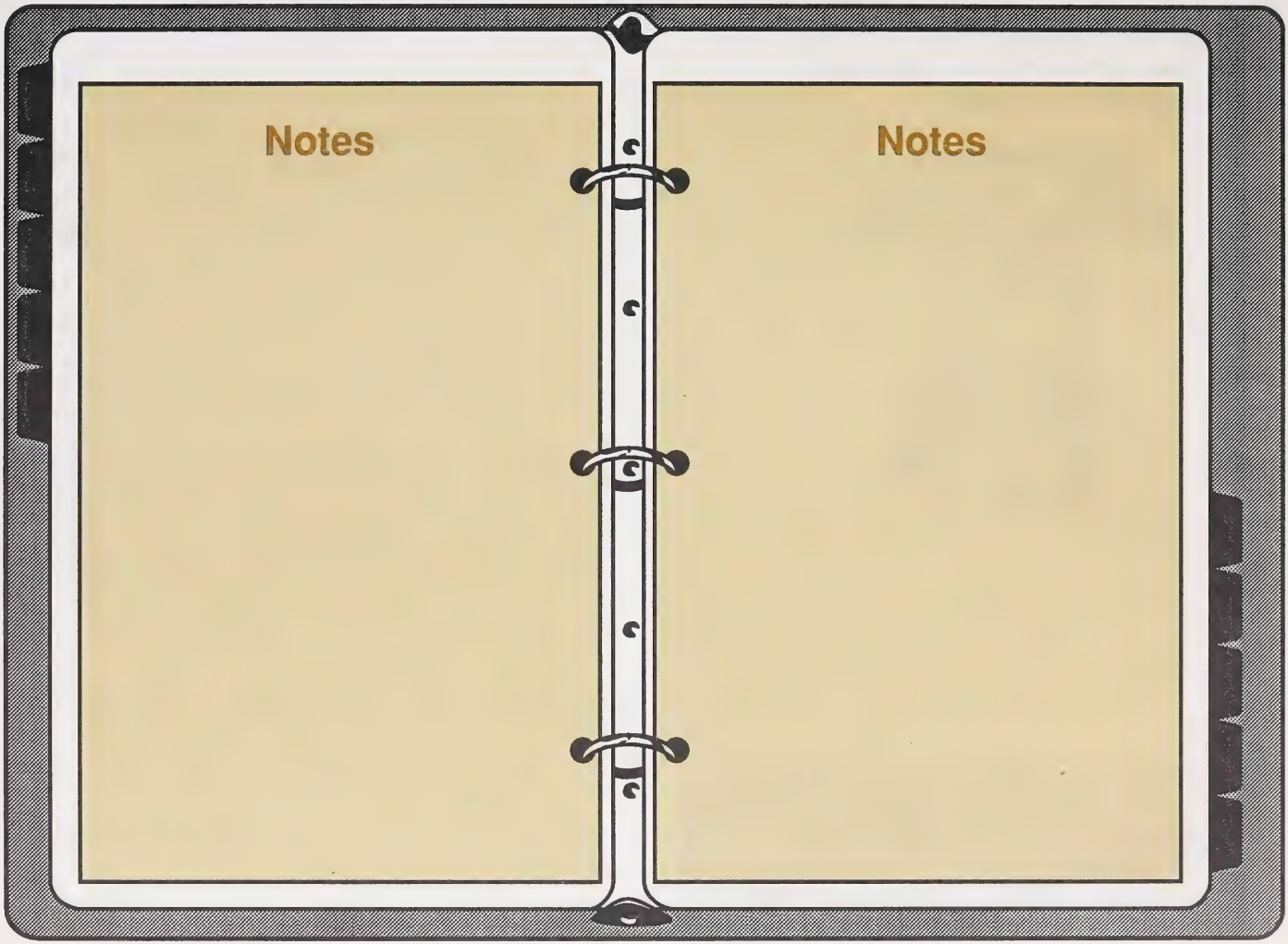
2. How many rectangles are in this figure?



3. A well is 10 m deep. A salamander climbs up 5 m during the day and climbs down 4 m at night. If the salamander started at the bottom on Monday, on what day will it get to the top?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

MODULE CONCLUSION

What Lies Ahead

In this section you will complete the module assignment.

Working Together

Now that you have studied Module 1 and you have done the required practice, you should be ready to do the assignment for Module 1.

Module Assignment

Turn to the Assignment Booklet and complete the assignment independently. You may refer to your notes, but do not get help from anyone.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

APPENDIX

GLOSSARY

Flow chart: a diagram that shows a sequence of events or instructions

Problem: a task for which the method for finding the answer (as well as the answer) is not immediately known

Tree diagram: a branching diagram used to list all the combinations

L.R.D.C.
Producer

MATH 8

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